

Command Manual and User Reference
for *OpenSees* Soil Models and Fully Coupled Element
Developed at University of California at San Diego

By
Zhaohui Yang (zhyang@ucsd.edu)
Ahmed Elgamal (elgamal@ucsd.edu)

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PressureDependMultiYield

DESCRIPTION

PressureDependMultiYield material is an elastic-plastic material for simulating the essential response characteristics of pressure sensitive soil materials under general loading conditions. Such characteristics include dilatancy (shear-induced volume contraction or dilation) and liquefaction (cyclic mobility), typically exhibited in sands or silts during monotonic or cyclic loading.

When this material is employed in regular solid elements (e.g., **FourNodeQuad**, **Brick**), it simulates drained soil response. To simulate soil response under fully undrained condition, this material may be either embedded in a **FluidSolidPorousMaterial** (see below), or used with the **FourNodeQuadUP** element (see below) with very low permeability. To simulate partially drained soil response, this material should be used with the **FourNodeQuadUP** element with proper permeability values.

During the application of gravity load (and static loads if any), material behavior is linear elastic. In the subsequent dynamic (fast) loading phase(s), the stress-strain response is elastic-plastic (see MATERIAL STAGE UPDATE below). Plasticity is formulated based on the multi-surface (nested surfaces) concept, with a non-associative flow rule to reproduce dilatancy effect. The yield surfaces are of the Drucker-Prager type.

For more information, see the references listed at the end of this manual.

INPUT INTERFACE:

```
nDMaterial PressureDependMultiYield tag? nd? rho? refShearModul? refBulkModul?  
frictionAng? peakShearStra? refPress? pressDependCoe? PTAng? contrac? dilat1?  
dilat2? liquefac1? liquefac2? liquefac3? <noYieldSurf?=20 < $\gamma_1$   $G_{s1}$  ...> e?=0.6  
cs1?=0.9 cs2?=0.02 cs3?=0.7  $p_a$ ?=101>
```

tag: A positive integer uniquely identifying the material among all *nDMaterials*.

nd: Number of dimensions, 2 for plane-strain, and 3 for 3D analysis.

rho: Saturated soil mass density.

refShearModul (G_r): Reference low-strain shear modulus, specified at a reference mean effective confining pressure *refPress* of p'_r (see below).

refBulkModul (B_r): Reference bulk modulus, specified at a reference mean effective confining pressure *refPress* of p'_r (see below).

frictionAng (ϕ): Friction angle at peak shear strength, in degrees.

peakShearStra (γ_{\max}): An octahedral shear strain at which the maximum shear strength is reached, specified at a reference mean effective confining pressure *refPress* of p'_r (see below).

Octahedral shear strain is defined as:

$$\gamma = \frac{2}{3} \left[(\varepsilon_{xx} - \varepsilon_{yy})^2 + (\varepsilon_{yy} - \varepsilon_{zz})^2 + (\varepsilon_{xx} - \varepsilon_{zz})^2 + 6\varepsilon_{xy}^2 + 6\varepsilon_{yz}^2 + 6\varepsilon_{xz}^2 \right]^{1/2}$$

refPress (p'_r): Reference mean effective confining pressure at which G_r , B_r , and γ_{\max} are defined.

pressDependCoe (d): A positive constant defining variations of G and B as a function of instantaneous effective confinement p' :

$$G = G_r \left(\frac{p'}{p'_r} \right)^d \quad B = B_r \left(\frac{p'}{p'_r} \right)^d$$

PTAng (ϕ_{PT}): Phase transformation angle, in degrees.

contrac: A non-negative constant defining the rate of shear-induced volume decrease (contraction) or pore pressure buildup. A larger value corresponds to faster contraction rate.

dilat1, *dilat2*: Non-negative constants defining the rate of shear-induced volume increase (dilation). Larger values correspond to stronger dilation rate.

liquefac1, *liquefac2*, *liquefac3*: Parameters controlling the mechanism of liquefaction-induced perfectly plastic shear strain accumulation, i.e., cyclic mobility (Yang 2000). **Set *liquefac1* = 0 to deactivate this mechanism altogether.** *liquefac1* defines the effective confining pressure (e.g., 10 kPa) below which the mechanism is in effect. Smaller values should be assigned to denser sands. *liquefac2* defines the maximum amount of perfectly plastic octahedral shear strain (e.g., 1.0) developed at zero effective confinement under monotonic shear loading condition. Smaller values should be assigned to denser sands. *liquefac3* defines the maximum amount of perfectly plastic octahedral shear strain γ_r accumulated at each loading cycle under biased shear loading condition, as the ratio to *liquefac2* (i.e., $\gamma_r = \text{liquefac2} \times \text{liquefac3}$). Typically *liquefac3* takes values between 0.0 and 3.0. Smaller values should be assigned to denser sands.

noYieldSurf: Number of yield surfaces, optional (must be less than 40, default is 20). The surfaces are generated based on the hyperbolic relation defined in Note 2 below.

γ , G_s : Instead of automatic surfaces generation (Note 2), **you can define yield surfaces directly based on desired shear modulus reduction curve.** To do so, add a minus sign in front of *noYieldSurf*, then provide *noYieldSurf* pairs of shear strain (γ) and modulus ratio (G_s) values. For example, to define 10 surfaces:

... -10 γ_1 G_{s_1} ... γ_{10} $G_{s_{10}}$...

See Note 3 below for some important notes.

e : Initial void ratio, optional (default is 0.6).

$cs1, cs2, cs3, p_a$: Parameters defining a straight critical-state line e_c in $e-p'$ space.

If $cs3=0$,

$$e_c = cs1 - cs2 \log(p'/p_a)$$

else (Li and Wang, JGGE, 124(12)),

$$e_c = cs1 - cs2(p'/p_a)^{cs3}$$

where p_a is atmospheric pressure for normalization (typically 101 kPa in SI units). All four constants are optional (default values: $cs1=0.9, cs2=0.02, cs3=0.7, p_a=101$).

Notes:

1. The friction angle ϕ defines the variation of peak (octahedral) shear strength τ_f as a function of current effective confinement p' :

$$\tau_f = \frac{2\sqrt{2} \sin \phi}{3 - \sin \phi} p'$$

Octahedral shear stress is defined as:

$$\tau = \frac{1}{3} \left[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6\sigma_{xy}^2 + 6\sigma_{yz}^2 + 6\sigma_{xz}^2 \right]^{1/2}$$

2. (Automatic surface generation) At a constant confinement p' , the shear stress τ (octahedral) - shear strain γ (octahedral) nonlinearity is defined by a hyperbolic curve (backbone curve):

$$\tau = \frac{G\gamma}{1 + \gamma/\gamma_r}$$

where γ_r satisfies the following equation at p'_r :

$$\tau_f = \frac{2\sqrt{2} \sin \phi}{3 - \sin \phi} p'_r = \frac{G_r \gamma_{\max}}{1 + \gamma_{\max}/\gamma_r}$$

3. (User defined surfaces) The user specified friction angle ϕ is ignored. Instead, ϕ is defined as follows:

$$\sin \phi = \frac{3\sqrt{3} \sigma_m / p'_r}{6 + \sqrt{3} \sigma_m / p'_r}$$

where σ_m is the product of the last modulus and strain pair in the modulus reduction curve. Therefore, it is important to adjust the backbone curve so as to render an appropriate ϕ . If the resulting ϕ is smaller than the phase transformation angle ϕ_{PT} , ϕ_{PT} is set equal to ϕ .

Also remember that improper modulus reduction curves can result in strain softening response (negative tangent shear modulus), which is not allowed in the current model

formulation. Finally, note that the backbone curve varies with confinement, although the variations are small within commonly interested confinement ranges. Backbone curves at different confinements can be obtained using the OpenSees element recorder facility (see OUTPUT INTERFACE below).

4. The last five optional parameters are needed when critical-state response (flow liquefaction) is anticipated. Upon reaching the critical-state line, material dilatancy is set to zero.

MATERIAL STAGE UPDATE:

A material stage (elastic or plastic) may be set by executing the following TCL command:

updateMaterialStage -material tag? -stage sNum?

where *tag* is the material number, and *sNum* is the desired stage:

0 - Linear elastic,

1 - Plastic,

2 - Linear elastic, with elasticity constants (shear modulus and bulk modulus) as a function of initial effective confinement.

To conduct a seismic analysis, two stages should be followed. First, during the application of gravity load (and static loads if any), set material stage to 0, and material behavior is linear elastic (with G_r and B_r as elastic moduli). After the application of gravity load, set material stage to 1 or 2. In case of stage 2, all the elastic material properties are then internally determined at the current effective confinement, and remain constant thereafter. In the subsequent dynamic (fast) loading phase(s), the deviatoric stress-strain response is elastic-plastic (stage 1) or linear-elastic (stage 2), and the volumetric response remains linear-elastic.

PARAMETER UPDATE

Currently, two of the material parameters, reference low-strain shear modulus G_r and reference bulk modulus B_r , can be modified during an analysis, by invoking the following TCL command:

updateParameter -material tag? -refG newVal?

or

updateParameter -material tag? -refB newVal?

where *tag* is the material number, and *newVal* is the new value.

OUTPUT INTERFACE:

The following information may be extracted for this material at a given integration point, using the OpenSees Element Recorder facility (McKenna and Fenves 2001): "**stress**", "**strain**", "**backbone**", or "**tangent**".

For 2D problems, the stress output follows this order: $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \eta_r$, where η_r is the ratio between the current shear (deviatoric) stress and peak shear strength at the current confinement ($0 \leq \eta_r \leq 1.0$). The strain output follows this order: $\epsilon_{xx}, \epsilon_{yy}, \gamma_{xy}$.

For 3D problems, the stress output follows this order: $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}, \eta_r$, and the strain output follows this order: $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}$.

The "**backbone**" option records (secant) shear modulus reduction curves at one or more given confinements. The specific recorder command is as follows:

```
recorder Element eleNum? -file fName? -dT deltaT? material intNum? backbone p1?
<p2? ...>
```

where p1, p2, ... are the confinements at which modulus reduction curves are recorded. In the output file, corresponding to each given confinement there are two columns: γ and secant modulus. The number of rows equals the number of yield surfaces.

SUGGESTED PARAMETER VALUES

For user convenience, a table is provided below as a quick reference for selecting parameter values. However, use of this table should be of great caution, and other information should be incorporated wherever possible.

	Loose Sand (15%-35%)	Medium Sand (35%-65%)	Medium-dense Sand (65%-85%)	Dense Sand (85%-100%)
<i>rho</i> (ton/m ³)	1.7	1.9	2.0	2.1
<i>refShearModul</i> (kPa, at $p'_r=80$ kPa)	5.5×10^4	7.5×10^4	1.0×10^5	1.3×10^5
<i>refBulkModu</i> (kPa, at $p'_r=80$ kPa)	1.5×10^5	2.0×10^5	3.0×10^5	3.9×10^5
<i>frictionAng</i>	29	33	37	40
<i>peakShearStra</i> (at $p'_r=80$ kPa)	0.1	0.1	0.1	0.1
<i>pressDependCoe</i>	0.5	0.5	0.5	0.5
<i>PTAng</i>	27	23	20	16
<i>contrac</i>	0.21	0.07	0.05	0.03
<i>dilat1</i>	0.	0.4	0.6	0.8
<i>dilat2</i>	0	2	3	5
<i>liquefac1</i> (kPa)	10	10	5	0
<i>liquefac2</i>	0.02	0.01	0.003	0
<i>liquefac3</i>	1	1	1	0
<i>e</i>	0.85	0.7	0.55	0.45

PressureIndependMultiYield

DESCRIPTION

PressureIndependMultiYield material is an elastic-plastic material in which plasticity exhibits only in the deviatoric stress-strain response. The volumetric stress-strain response is linear-elastic and is independent of the deviatoric response. This material is implemented to simulate monotonic or cyclic response of materials whose shear behavior is insensitive to the confinement change. Such materials include, for example, organic soils or clay under fast (undrained) loading conditions.

During the application of gravity load (and static loads if any), material behavior is linear elastic. In the subsequent dynamic (fast) loading phase(s), the stress-strain response is elastic-plastic (see MATERIAL STAGE UPDATE below). Plasticity is formulated based on the multi-surface (nested surfaces) concept, with an associative flow rule. The yield surfaces are of the Drucker-Prager type.

For more information, see the references listed at the end of this manual.

INPUT INTERFACE:

```
nDMaterial PressureIndependMultiYield tag? nd? rho? refShearModul?  
refBulkModul? cohesi? peakShearStra? <frictionAng?=0. refPress?=100.  
pressDependCoe?=0. noYieldSurf?=20 < $\gamma_1$ ?  $G_1$ ? ...> >
```

tag: A positive integer uniquely identifying the material among all *nDMaterials*.

nd: Number of dimensions, 2 for plane-strain, and 3 for 3D analysis.

refShearModul (G_r): Reference low-strain shear modulus, specified at a reference mean effective confining pressure *refPress* of p'_r (see below).

refBulkModul (B_r): Reference bulk modulus, specified at a reference mean effective confining pressure *refPress* of p'_r (see below).

cohesi (c): Apparent cohesion at zero effective confinement.

peakShearStra (γ_{\max}): An octahedral shear strain at which the maximum shear strength is reached, specified at a reference mean effective confining pressure *refPress* of p'_r (see below).

frictionAng (ϕ): Friction angle at peak shear strength in degrees, optional (default is 0.0).

refPress (p'_r): Reference mean effective confining pressure at which G_r , B_r , and γ_{\max} are defined, optional (default is 100.).

pressDependCoe (*d*): An optional non-negative constant defining variations of *G* and *B* as a function of initial effective confinement p'_i (default is 0.0):

$$G = G_r \left(\frac{p'_i}{p'_r} \right)^d \quad B = B_r \left(\frac{p'_i}{p'_r} \right)^d$$

noYieldSurf: Number of yield surfaces, optional (must be less than 40, default is 20). The surfaces are generated based on the hyperbolic relation defined in Note 2 below.

γ , *G_s*: Instead of automatic surfaces generation (Note 2), **you can define yield surfaces directly based on desired shear modulus reduction curve**. To do so, add a minus sign in front of *noYieldSurf*, then provide *noYieldSurf* pairs of shear strain (γ) and modulus ratio (*G_s*) values. For example, to define 10 surfaces:

... -10 γ_1 *G_{s1}* ... γ_{10} *G_{s10}*

See Note 3 below for some important notes.

Notes:

1. The friction angle ϕ and cohesion *c* define the variation of peak (octahedral) shear strength τ_f as a function of initial effective confinement p'_i :

$$\tau_f = \frac{2\sqrt{2} \sin \phi}{3 - \sin \phi} p'_i + \frac{2\sqrt{2}}{3} c$$

2. Automatic surface generation: at a constant confinement p' , the shear stress τ (octahedral) - shear strain γ (octahedral) nonlinearity is defined by a hyperbolic curve (backbone curve):

$$\tau = \frac{G\gamma}{1 + \gamma / \gamma_r}$$

where γ_r satisfies the following equation at p'_r :

$$\tau_f = \frac{2\sqrt{2} \sin \phi}{3 - \sin \phi} p'_r + \frac{2\sqrt{2}}{3} c = \frac{G_r \gamma_{\max}}{1 + \gamma_{\max} / \gamma_r}$$

3. (User defined surfaces) If the user specifies $\phi=0$, cohesion *c* will be ignored. Instead, *c* is defined as $c = \sqrt{3} \sigma_m / 2$, where σ_m is the product of the last modulus and strain pair in the modulus reduction curve. Therefore, it is important to adjust the backbone curve so as to render an appropriate *c*.

If the user specifies $\phi > 0$, this ϕ will be ignored. Instead, ϕ is defined as follows:

$$\sin \phi = \frac{3(\sqrt{3} \sigma_m - 2c) / p'_r}{6 + (\sqrt{3} \sigma_m - 2c) / p'_r}$$

If the resulting $\phi < 0$, we set $\phi=0$ and $c = \sqrt{3} \sigma_m / 2$.

Also remember that improper modulus reduction curves can result in strain softening response (negative tangent shear modulus), which is not allowed in the current model

formulation. Finally, note that the backbone curve varies with confinement, although the variation is small within commonly interested confinement ranges. Backbone curves at different confinements can be obtained using the OpenSees element recorder facility (see OUTPUT INTERFACE below).

MATERIAL STAGE UPDATE:

A material stage (elastic or plastic) may be set by executing the following TCL command:

updateMaterialStage -material tag? -stage sNum?

where *tag* is the material number, and *sNum* is the desired stage:

- 0 - Linear elastic,
- 1 - Plastic,
- 2 - Linear elastic, with elasticity constants (shear modulus and bulk modulus) as a function of initial effective confinement.

To conduct a seismic analysis, two stages may be followed. First, during the application of gravity load (and static loads if any), set material stage to 0, and material behavior is linear elastic (with G_r and B_r as elastic moduli). After the application of gravity load, set material stage to 1 or 2. All the elastic/plastic material properties are then internally determined at the current effective confinement, and remain constant thereafter. In the subsequent dynamic (fast) loading phase(s), the deviatoric stress-strain response is elastic-plastic (stage 1) or linear-elastic (stage 2), and the volumetric response remains linear-elastic.

PARAMETER UPDATE

Same as that for the **PressureIndependMultiYield** material above.

OUTPUT INTERFACE:

Same as that for the **PressureIndependMultiYield** material above.

SUGGESTED PARAMETER VALUES

For user convenience, a table is provided below as a quick reference for selecting parameter values. However, use of this table should be of great caution, and other information should be incorporated wherever possible.

	Soft Clay	Medium Clay	Stiff Clay
<i>rho</i> (ton/m ³)	1.3	1.5	1.8
<i>refShearModul</i> (kPa)	1.3x10 ⁴	6.0x10 ⁴	1.5x10 ⁵
<i>refBulkModu</i> (kPa)	6.5x10 ⁴	3.0x10 ⁵	7.5x10 ⁵
<i>cohesi</i> (kPa)	18	37	75
<i>peakShearStra</i>	0.1	0.1	0.1
<i>frictionAng</i>	0	0	0
<i>pressDependCoe</i>	0	0	0

FluidSolidPorousMaterial

DESCRIPTION

FluidSolidPorousMaterial couples the responses of two phases: fluid and solid. The fluid phase response is only volumetric and linear elastic. The solid phase can be any NDMaterial. This material is developed to simulate the response of saturated porous media under fully undrained condition.

INPUT INTERFACE

nDMaterial **FluidSolidPorousMaterial** tag? nd? soilMatTag? combinedBulkModul?

tag: A positive integer uniquely identifying the material among all *nDMaterials*.

nd: Number of dimensions, 2 for plane-strain, and 3 for general 3D analysis.

soilMatTag: The material number for the solid phase material (previously defined).

combinedBulkModul: Combined undrained bulk modulus B_c relating changes in pore pressure and volumetric strain, may be approximated by:

$$B_c \approx B_f / n$$

where B_f is the bulk modulus of fluid phase (2.2×10^6 kPa for water typically), and n the initial porosity.

Notes:

1. Buoyant unit weight (total unit weight - fluid unit weight) should be used in definition of the finite elements composed of a **FluidSolidPorousMaterial**.
2. During the application of gravity (elastic) load (i.e., material stage 1, see below), the fluid phase does not contribute to the material response.

MATERIAL STAGE UPDATE:

A material stage (elastic or plastic) may be set by executing the following TCL command:

updateMaterialStage -material tag? -stage sNum?

where *tag* is the material number, and *sNum* is the desired stage:

- 0 - Linear elastic,
- 1 - Plastic,
- 2 - Linear elastic, with elasticity constants of embedded soil material (shear modulus and bulk modulus) as a function of initial effective confinement.

To conduct a seismic analysis, two stages should be followed. First, during the application of gravity load (and static loads if any), set material stage to 0. The fluid phase does not contribute

to the material response at this stage. Before the subsequent dynamic (fast) loading phase(s), change the material stage to 1 or 2.

OUTPUT INTERFACE:

The following information may be extracted for this material at given integration point, using the OpenSees Element Recorder facility (McKenna and Fenves 2001): "**stress**", "**strain**", "**tangent**", or "**pressure**". The "**pressure**" option records excess pore pressure and excess pore pressure ratio at a given material integration point.

FourNodeQuadUP

DESCRIPTION

FourNodeQuadUP is a four-node plane-strain element using bilinear isoparametric formulation. This element is implemented for simulating dynamic response of solid-fluid fully coupled material, based on Biot's theory of porous medium. Each element node has 3 degrees-of-freedom (DOF): DOF 1 and 2 for solid displacement (u) and DOF 3 for fluid pressure (p).

For more information, see the references listed at the end of this manual.

INPUT INTERFACE

element **quadUP** eleTag? iNode? jNode? kNode? lNode? thick? type? matTag?
bulk? fmass? hPerm? vPerm? <b1?=0 b2?=0 t?=0>

eleTag: A positive integer uniquely identifying the element among all *elements*.

iNode, *jNode*, *kNode*, *lNode*: Four element node (previously defined) numbers in counter-clockwise order around the element.

thick: Element thickness.

type: The string "PlaneStrain".

matTag: Tag of an NDMaterial object (previously defined) of which the element is composed.

bulk: Combined undrained bulk modulus B_c relating changes in pore pressure and volumetric strain, may be approximated by:

$$B_c \approx B_f / n$$

where B_f is the bulk modulus of fluid phase (2.2×10^6 kPa for water), and n the initial porosity.

fmass: Fluid mass density.

hPerm: Permeability coefficient in horizontal direction.

vPerm: Permeability coefficient in vertical direction.

b1, *b2*: Optional body forces in horizontal and vertical directions respectively (defaults are 0.0).

t: Optional uniform element normal traction, positive in tension (default is 0.0).

OUTPUT INTERFACE:

Pore pressure can be recorded at an element node using OpenSees Node Recorder:

```
recorder Node fileName? vel -time -node nod1? nod2? ... -dof 3
```

See OpenSees command manual (McKenna and Fenves 2001) for nodal displacement, velocity, or acceleration recorders.

The valid queries to a quadUP element when creating an ElementRecorder are 'force', 'stiffness', or 'material *matNum matArg1 matArg2 ...*', where *matNum* represents the material object at the corresponding integration point.

TYPICAL RANGE OF PERMEABILITY COEFFICIENT (m/s)

Gravel	Sand	Silty Sand	Silt	Clay
$>1.0 \times 10^{-3}$	$1.0 \times 10^{-5} \sim 1.0 \times 10^{-3}$	$1.0 \times 10^{-7} \sim 1.0 \times 10^{-5}$	$1.0 \times 10^{-9} \sim 1.0 \times 10^{-7}$	$<1.0 \times 10^{-9}$

References

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