17th ASCE Engineering Mechanics Conference
June 13–16, 2004, University of Delaware, Newark, DE
EM2004

A MULTI-SURFACE PLASTICITY SAND MODEL INCLUDING THE LODE ANGLE EFFECT

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ABSTRACT

A multi-surface plasticity model for cohesionless granular materials is developed, in which the Lade-Duncan failure criterion is employed as the yield function. This yield function includes the first and third stress invariants to account for the dependence of cyclic shear stress-strain behavior on confining pressure and the Lode angle. A purely deviatoric kinematic hardening rule is employed to generate hysteretic response under arbitrary cyclic loading conditions. Associativity of plastic flow is followed in the deviatoric plane. A non-associative flow rule is formulated for the hydrostatic component. Salient features of the model performance are presented under general three-dimensional (3D) loading conditions.

Keywords: Lode angle, cyclic plasticity, multi-surface plasticity, soil liquefaction, earthquake, soil dynamics

INTRODUCTION

The shear strength of granular soils strongly depends on confining pressure and the Lode angle effect (e.g., Lade and Duncan 1973, Chen 1982, Yamada and Ishihara 1983, Peric and Ayari 2002, Gutta et al. 2003). In soil plasticity models, this dependence may be represented by a yield (or failure) function that includes appropriate stress invariants (e.g., Argyris et al. 1974, Willam and Warnke 1974, Matsuoka 1974, Lade and Duncan 1975, Lade 1977, Ottosen 1977). For instance, the Lade-Duncan failure criterion (1975) was employed to model a large body of laboratory sample test data on cohesionless soils (Fig. 1). This criterion is defined by:

$$I_1^3 / I_3 = \kappa_1 \qquad (1)$$

where I_1 and I_3 are the first and third stress invariants respectively, and κ_1 (>27) is a parameter related to soil shear strength (or friction angle ϕ).

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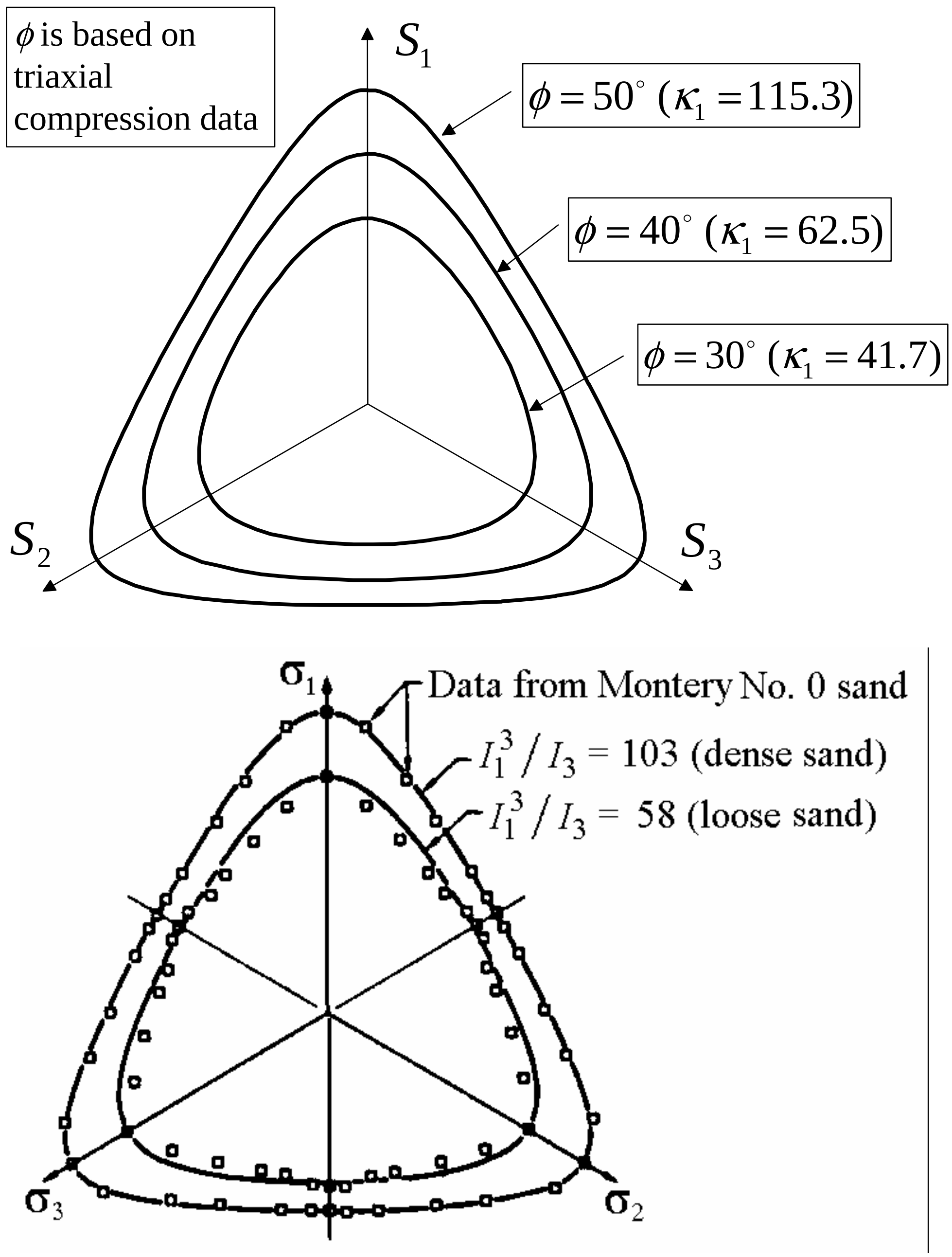

Fig. 1. Lade-Duncan failure criterion for various friction angles and experimental verification (Lade and Duncan 1975).

Failure criteria involving the Lode angle have been implemented in a large number of soil constitutive models, including bounding surface models (e.g., Dafalias 1986, Anandarajah and Dafalias 1986, Kaliakin et al. 1990, Wang et al. 1990, Anandarajah 1994, Manzari and Dafalias 1997, Li 1997, Manzari and Nour 2000, Li and Dafalias 2000, Ling et al. 2002), rotational hardening models (e.g., Inel and Lade 1997, Gutta et al. 2003), generalized plasticity models (e.g., Ling and Liu 2003), as well as isotropic hardening models (e.g., Lade and Duncan 1975, Lade and Kim 1988, Peric and Ayari 2002).

In this paper, a multi-surface plasticity model is described, in which the Lade-Duncan failure criterion (Eq. 1) is chosen as the yield function. The model formulation is first presented, including the employed yield function, hardening rule, and flow rule. Thereafter, a number of numerical simulations (drained and undrained) are shown to demonstrate the Lode angle effects under general 3D loading conditions.

MODEL FORMULATION

The model is based on the J_2-plasticity framework developed by Prevost (1985) for granular materials, in which a multi-surface approach was adopted (Iwan 1967, Mroz 1967). The constitutive equation is written in incremental form as (Prevost 1985):

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p) \tag{2}$$

where $\dot{\boldsymbol{\sigma}}$ is the rate of effective stress tensor, $\dot{\boldsymbol{\varepsilon}}$ is the rate of deformation tensor, $\dot{\boldsymbol{\varepsilon}}^p$ is the plastic rate of deformation tensor, and **E** is the isotropic fourth-order tensor of elastic coefficients. In this equation a superposed dot denotes material time derivative. The plastic rate of deformation tensor is defined by: $\dot{\boldsymbol{\varepsilon}}^p = \mathbf{P} \left\langle L \right\rangle$, where **P** is a symmetric second-order tensor defining the outer normal to the plastic potential surface, *L* is the plastic loading function; and the symbol $\left\langle \ \right\rangle$ denotes the McCauley's brackets (i.e., *L*=0 if *L*<0). In the above, *L* is defined as: $L = (\mathbf{Q} : \dot{\boldsymbol{\sigma}}) / H'$ where H' is the plastic modulus, and **Q** a unit symmetric second-order tensor defining the outer normal to the yield surface.

The yield function (Eq. 1) forms a smooth conical surface in principal stress space (Fig. 2), with apex at the origin. In the context of multi-surface plasticity, a number of similar yield surfaces with a common apex and different sizes form the hardening zone (Fig. 2). Each surface is associated with a constant plastic modulus.

A purely deviatoric kinematic hardening rule is employed (Mroz 1967, Prevost 1985). Thus, all surfaces but the outermost may translate in stress space. The direction of translation may be selected independently of any formal plasticity constraints, and is generally governed by the consideration that no overlapping is allowed between yield surfaces (Mroz 1967). In this model, the direction of surface translation is modified from the original Mroz's rule for enhanced computational efficiency (Elgamal et al. 2003). With the direction of translation defined, the amount of translation may then be obtained by satisfying the consistency condition $\dot{f} = 0$.

The flow rule follows closely that developed earlier for a multi-surface model with the Drucker-Prager yield function (Yang et al. 2003). This rule was developed to reproduce salient response mechanisms associated with liquefaction and the resulting shear deformations. We decompose the outer normals to the yield surface and the plastic potential surface (**Q** and **P**) into deviatoric and volumetric components giving $\mathbf{Q} = \mathbf{Q}' + \mathrm{Q}'' \boldsymbol{\delta}$ and $\mathbf{P} = \mathbf{P}' + \mathrm{P}'' \boldsymbol{\delta}$. The associated flow is prescribed in the deviatoric plane, and nonassociativity of plastic flow is restricted to the volumetric component (Prevost 1985), i.e., $\mathbf{Q'} = \mathbf{P'}$ and $\mathrm{P''} \neq \mathrm{Q''}$.

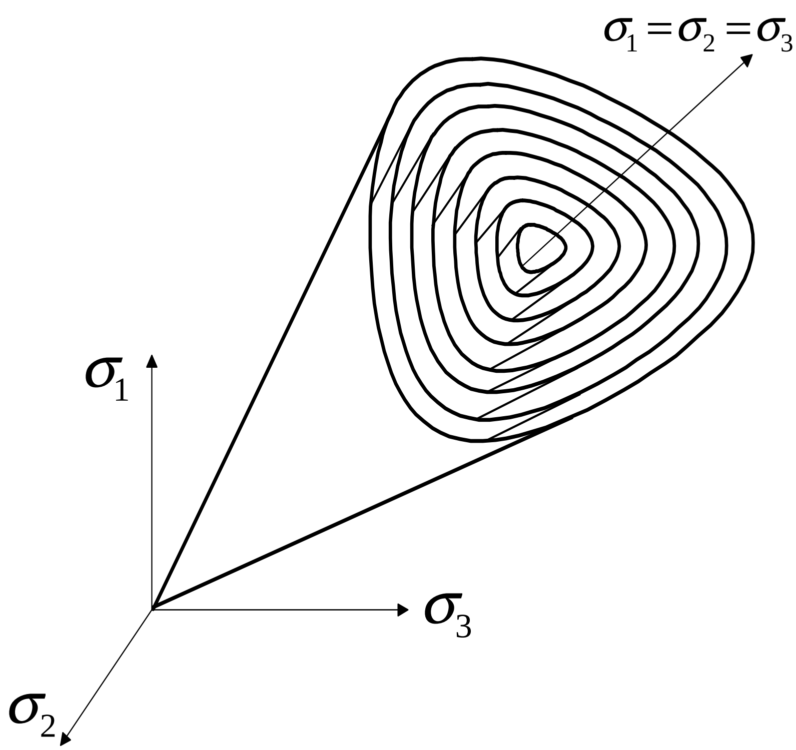

Fig. 2. Configuration of multi Lade-Duncan yield surfaces in principal stress space.

MODEL PERFORMANCE

A major advantage of including J_3 over J_2-only models is added accuracy in modeling soil deformation under general three-dimensional (3D) loading scenarios. Herein, a number of 3D model simulation results illustrate some salient response characteristics.

Using discrete element numerical simulation, Thornton (2000) obtained the response of a system of elastic spheres subjected to a constant-deviatoric strain path (a circle in deviatoric strain space), with constant mean effective confinement. The resulting deviatoric stress path, along with that simulated by our constitutive model, is shown in Fig. 3. This non-circular stress path is attributed to the Lode angle effect, since a J_2 function would result in a perfectly circular stress path.

In Fig. 4, the result of a laboratory experiment is shown (Lewin 1980), where soil was subjected to a constant-deviatoric stress path (a circle in deviatoric stress space), under drained conditions. In this test, the applied mean effective confining pressure remained unchanged throughout. The resulting deviatoric strain path is shown in Fig. 4a, along with the strain path simulated by our model (Fig. 4b). Both paths are of the same form as the stress paths in the last case (Figs. 3a and 3b), and manifest the Lode angle effect. A result similar to that of Fig. 4b was also reported by Dafalias and Herrmann (1986).

ACKNOWLEDGMENTS

This work was supported by the Pacific Earthquake Engineering Research (PEER) Center, under the National Science Foundation (Award No. EEC-9701568), and by NSF Grants (No. CMS0084616 and No. CMS0200510). Professor Paul V. Lade (Catholic University of America, Washington D.C.) encouraged us to conduct this research. Professor Jerry A. Yamamuro (University of Delaware) and Professor Xiang-Song Li (Hong Kong University of Science and Technology) kindly provided valuable suggestions and insights. The authors are most grateful for this assistance.

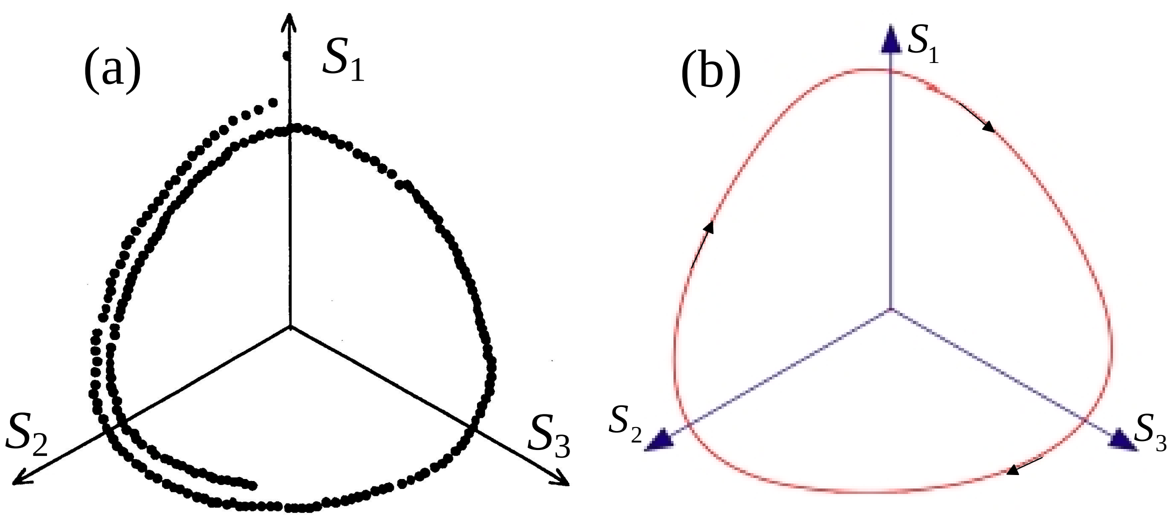

Fig. 3. Imposed constant deviatoric-strain path under drained, constant mean confinement conditions: (a) resulting devistoric stress path using discrete-element simulation (Thornton, 2000), and (b) result from the developed model.

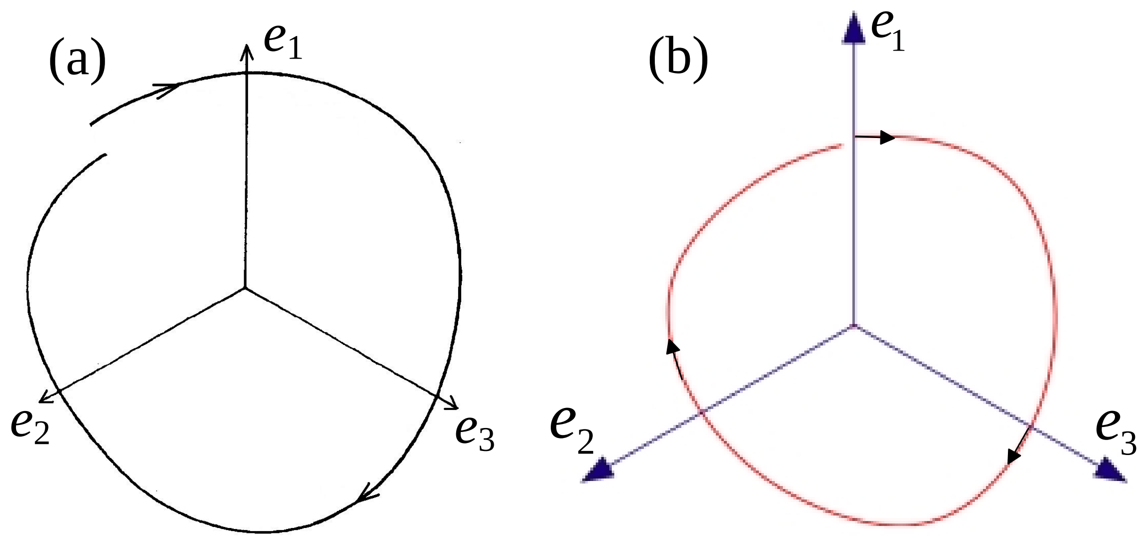

Fig. 4. Imposed constant deviatoric-stress path under drained, constant mean confinement conditions: (a) resulting devistoric strain path from laboratory sample test (after Lewin 1980), and (b) result from the developed model (e_1, e_2, and e_3 are principal deviatoric strains).

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